

Svolgimento compito matematica del 18-10-2015

1) Risolvere la seguente disequazione

$$ax - (1-a)(x+1) > (a-1)(a+1) + x$$

$$Ax + B > 0$$

$$ax - x - 1 + a + x + a > a^2 - 1 + x$$

$$2ax - 2x + a - a^2 > 0$$

$$2x(a-1) + a(1-a) > 0$$

$$2x(a-1) > a(a-1) \quad (*)$$

$$a-1 < 0 \quad a-1 = 0 \quad a-1 > 0$$

• Se $a-1 > 0 \Rightarrow a > 1$

$$\frac{2x(a-1)}{2(a-1)} > \frac{a(a-1)}{2(a-1)}$$

$$x > \frac{a}{2}$$

• Se $a-1 = 0 \Rightarrow a = 1$ sostituendo al posto di a il valore 1 nella (*) otteniamo $0 > 0$ impossibile

• Se $a-1 < 0 \Rightarrow a < 1$

$$2x(1-a) < a(1-a)$$

$$\parallel \parallel$$

$$-2x(a-1) \quad -a(a-1)$$

$$\frac{2x(1-a)}{2(1-a)} < \frac{a(1-a)}{2(1-a)}$$

$$x < \frac{a}{2}$$

2) $x^3 - 6x^2 + 5x + 12 < 0$

RUFFINI

$$\begin{array}{r|rrr|r} x-1 & 1 & -6 & 5 & 12 \\ x+1=0 & -1 & \downarrow & -1 & +7 & -12 \\ \hline & 1 & -7 & +12 & & \end{array} \quad \parallel \rightarrow \text{reste}$$

$$x^3 - 6x^2 + 5x + 12 = (x+1)(x^2 - 7x + 12) < 0$$

$$\begin{array}{c} x+1 > 0 \\ x^2 - 7x + 12 > 0 \end{array} \quad \begin{array}{c} - \\ + \\ + \\ + \\ + \\ + \\ + \\ + \end{array} \quad \begin{array}{c} + \\ + \\ + \\ + \\ + \\ + \\ + \\ + \end{array} \quad \begin{array}{c} x \\ x \\ x \\ x \\ x \\ x \\ x \\ x \end{array}$$

$$x_{1/2} = \frac{7 \pm \sqrt{49 - 48}}{2} = \frac{7 \pm 1}{2} = \begin{array}{l} 4 \\ 3 \end{array} \quad \begin{array}{c} - \\ + \\ + \\ - \end{array} \quad \begin{array}{c} x < -1 \\ 3 < x < 4 \end{array}$$

$$3) \frac{2}{1-x} + \frac{12}{x^2-1} > 1 + \frac{3}{x+1}$$

$$\frac{-2}{x-1} + \frac{12}{(x-1)(x+1)} > 1 + \frac{3}{x+1}$$

$$\frac{-2(x+1) + 12 - 1(x^2-1) - 3(x-1)}{x^2-1} > 0$$

$$\frac{-2x-2+12-x^2+1-3x+3}{x^2-1} > 0$$

$$\frac{-x^2 - 5x + 14}{x^2-1} > 0$$

4) $-x^2 - 5x + 14 > 0$

$$x_{1/2} = \frac{5 \pm \sqrt{25+56}}{-2} = \frac{5 \pm \sqrt{81}}{-2}$$

$$= \frac{5 \pm 9}{-2} = \begin{array}{l} -7 \\ +2 \end{array}$$

$$\begin{array}{c} - \\ + \\ - \end{array} \quad \begin{array}{c} x \\ x \\ x \end{array}$$

$$5) x^2 - 1 > 0 \quad \begin{array}{c} -1 & 1 \\ + & - \\ 1 & + \end{array} \quad \begin{array}{c} x \\ x \end{array}$$

$$\begin{array}{c} -7 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{array} \quad \begin{array}{c} x \\ x \\ x \\ x \end{array}$$

$$4) \left| \frac{3-2x}{x+6} \right| > 1$$

$$\frac{3-2x}{x+6} < -1 \cup \frac{3-2x}{x+6} > 1$$

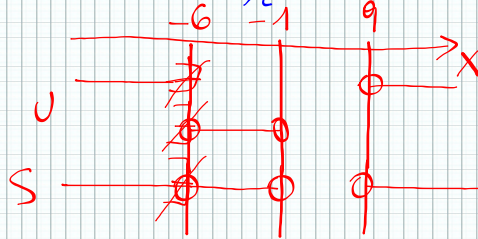
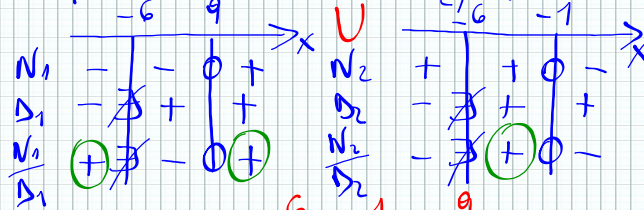
$$\frac{3-2x+x+6}{x+6} < 0 \cup \frac{3-2x-x-6}{x+6} > 0$$

$$\frac{-x+9}{x+6} < 0 \cup \frac{-3x-3}{x+6} > 0$$

$$\frac{x-9}{x+6} > 0 \cup \frac{-3x-3}{x+6} > 0$$

$$N_1: x-9 > 0 \Rightarrow x > 9 \cup N_2: -3x-3 > 0 \Rightarrow x < -1$$

$$D_1: x+6 > 0 \Rightarrow x > -6 \cup D_2: x+6 > 0 \Rightarrow x > -6$$



$$x < -1 \cup x > -6 \cup x > 9$$

$$5) \sqrt{16x-2} + 2 + 4x > 0$$

$$\sqrt{16x-2} > -2-4x$$

$$\begin{cases} -2-4x < 0 \\ 16x-2 > 0 \end{cases} \cup \begin{cases} -2-4x > 0 \\ 16x-2 > (-2-4x)^2 \end{cases}$$

$$\begin{cases} x > -\frac{1}{2} \\ x > \frac{1}{8} \end{cases} \cup \begin{cases} x < -\frac{1}{2} \\ 16x-2 > 4+16x^2+16x \end{cases}$$

$$\Downarrow \Downarrow \begin{cases} x > \frac{1}{8} \\ x < -\frac{1}{2} \\ 16x^2+6 < 0 \end{cases}$$

$\Downarrow \emptyset$

$$S: x > \frac{1}{8}$$

PROBLEMA

a) $|1-x^2| \leq 3 + |x-2|$

	-1	1	2	$\rightarrow x$
$1-x^2 \geq 0$	-	+	-	-
$x-2 \geq 0$	-	-	-	+

$$\begin{cases} x < -1 \cup 1 \leq x < 2 \\ -1+x^2 \leq 3-x+2 \end{cases} \cup \begin{cases} -1 \leq x < 1 \\ 1-x^2 \leq 3-x+2 \end{cases} \cup \begin{cases} x \geq 2 \\ -1+x^2 \leq 3+x-2 \end{cases}$$

$$\begin{cases} x < -1 \cup 1 \leq x < 2 \\ x^2+x-6 \leq 0 \end{cases} \cup \begin{cases} -1 \leq x < 1 \\ -x^2+x-4 \leq 0 \end{cases} \cup \begin{cases} x \geq 2 \\ x^2-x-2 \leq 0 \end{cases}$$

$$\begin{cases} x < -1 \cup 1 \leq x < 2 \\ -3 \leq x \leq 2 \end{cases} \cup \begin{cases} -1 \leq x < 1 \\ \forall x \in \mathbb{R} \end{cases} \cup \begin{cases} x \geq 2 \\ -1 \leq x \leq 2 \end{cases}$$

$$x_{1,2} = \frac{-1 \pm \sqrt{1+24}}{2} = \frac{-1 \pm 5}{2} < -3 \quad x_{3,4} = \frac{-1 \pm \sqrt{-16}}{-2}$$

$$x_{5,6} = \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm 3}{2} < -1$$

$\overline{AB} = 4 \text{ cm}$

b) $\sqrt{x+2} - \sqrt{x-3} \geq -\sqrt{x-5}$

$$\begin{cases} x+2 \geq 0 \rightarrow x \geq -2 \\ x-3 \geq 0 \rightarrow x \geq 3 \\ x-5 \geq 0 \rightarrow x \geq 5 \end{cases}$$

$$\sqrt{x+2+x-3-2} \sqrt{(x+2)(x-3)} \geq \sqrt{x-5}$$

$$\begin{cases} x \geq 5 \\ x+4 \geq 2\sqrt{(x+2)(x-3)} \end{cases}$$

$$\sqrt{A(x)} \leq B(x)$$

$$\begin{cases} x \geq 5 \\ x+4 \geq 0 \rightarrow x \geq -4 \\ (x+2)(x-3) \geq 0 \rightarrow x \leq -2 \cup x \geq 3 \\ x^2+16+8x \geq 4(x^2-x-6) \end{cases}$$

$$x^2+16+8x \geq 4x^2-4x-24$$

$$-3x^2+12x+40 \geq 0$$

$$x_{1,2} = \frac{-6 \pm \sqrt{36+120}}{-3} = \frac{-6 \pm \sqrt{156}}{-3}$$

$$= \begin{cases} -6 - \sqrt{156} = 6 + \sqrt{156} > 0 \\ -6 + \sqrt{156} = 6 - \sqrt{156} < 0 \end{cases}$$

$$\frac{6-\sqrt{156}}{3} \leq x \leq \frac{6+\sqrt{156}}{3}$$

$\overline{BC} = 5 \text{ cm}$

$$50 = (a^2+b^2)(c^2+d^2) = \dots$$

$$50 = 5 \times 10 = (4+1) \times (9+1) = (2^2+1^2) \times (3^2+1^2)$$

