

# IPERBOLE EQUILATERA TRASLATA: FUNZIONE OMOGRAFICA

$$y = \frac{ax+b}{cx+d}$$

con  $a, b, c, d \in \mathbb{R}$

• Se  $c=0$   $y = \frac{ax+b}{d}$

$y = \frac{a}{d}x + \frac{b}{d}$  retta

• se  $c \neq 0$   $ad-bc=0$  cioè  $ad=bc$

$$y = \frac{d \left( \frac{a}{d}x + \frac{b}{d} \right)}{b \left( \frac{c}{b}x + \frac{d}{b} \right)}$$

$$y = \frac{\frac{1}{d}(adx+bd)}{\frac{1}{b}(bcx+db)}$$

$$y = \frac{b}{d} \left[ \frac{bcx+bd}{bcx+db} \right]$$

$$y = \frac{b}{d} \left[ \frac{b(cx+d)}{b(cx+d)} \right]$$

$y = \frac{b}{d}$  retta

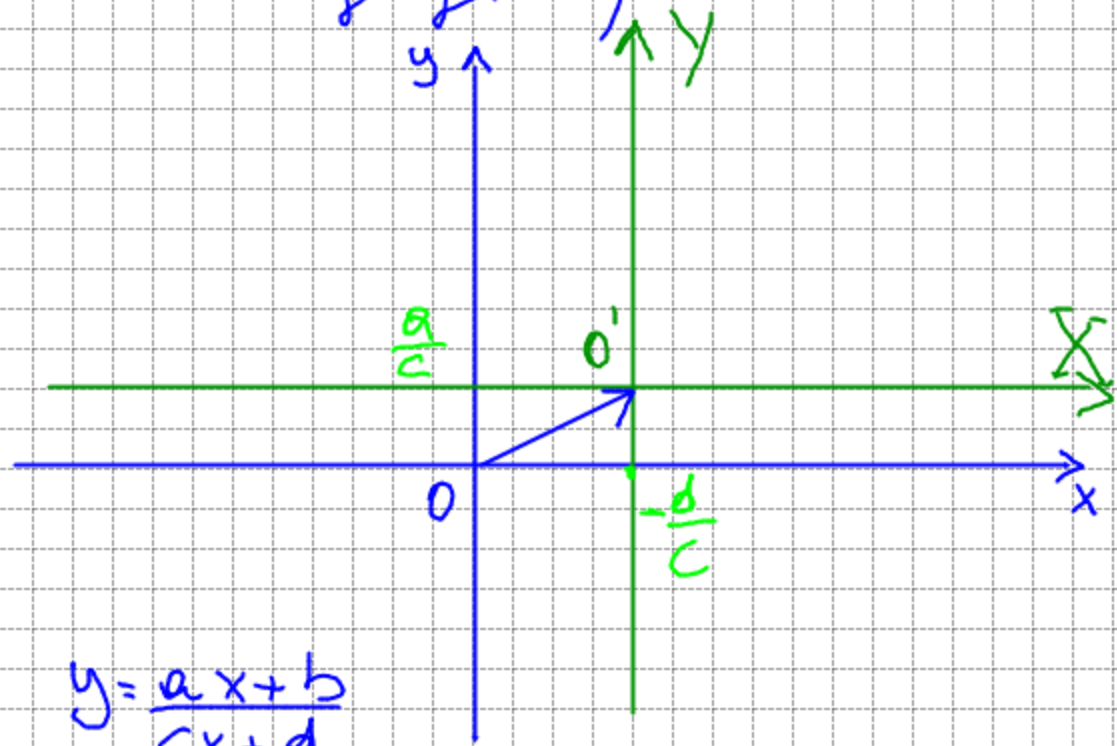
• se  $c \neq 0$  e  $ad-bc \neq 0$  otteniamo una iperbole equilatera

traslata (ovvero funzione omografica) di centro  $O' \left( -\frac{d}{c}; \frac{a}{c} \right)$

$$\begin{cases} X = x + \frac{d}{c} \\ Y = y - \frac{a}{c} \end{cases}$$

$$\begin{cases} x = X - \frac{d}{c} \\ y = Y + \frac{a}{c} \end{cases}$$

$$y = \frac{ax+b}{cx+d}$$



$$Y + \frac{a}{c} = \frac{a \left( X - \frac{d}{c} \right) + b}{c \left( X - \frac{d}{c} \right) + d}$$

$$Y + \frac{a}{c} = \frac{aX - \frac{da}{c} + b}{cX - \frac{d}{c} + d}$$

$$cXY = \cancel{aX} - \frac{da}{c} + b - \cancel{aX}$$

$$XY = \frac{b}{c} - \frac{ad}{c^2} = k$$

$$XY = k$$

## ESEMPIO

Determina equazione dell'iperbole equilatera traslata avente per asintoto  $x = -3$  e passante per  $O(0,0)$   $P(1, \frac{1}{4})$

$$y = \frac{ax+b}{cx+d}$$

$$c \neq 0 \quad ad - bc \neq 0$$

$$y = \frac{a(\frac{a}{c}x + \frac{b}{c})}{a(x + \frac{d}{c})}$$

$$\frac{a}{c} = A \quad \frac{b}{c} = B$$

$$\frac{d}{c} = D$$

$$y = \frac{Ax+B}{x+D}$$

$$x = -3 \Rightarrow cx+d=0 \Rightarrow x = -\frac{d}{c} \Rightarrow x = -D$$

$$P \begin{cases} -D = -3 \\ 0 = \frac{B}{D} \\ \frac{1}{4} = \frac{A+B}{D+1} \end{cases}$$

$$\begin{cases} D = 3 \\ B = 0 \\ \frac{1}{4} = \frac{A}{4} \end{cases} \quad \begin{cases} D = 3 \\ B = 0 \\ A = 1 \end{cases}$$

